

Standard Model Higgs field and energy scale of gravity

F.R. Klinkhamer # ¹⁾

Institute for Theoretical Physics, Karlsruhe Institute of Technology (KIT), 76128 Karlsruhe, Germany

The effective potential of the Higgs scalar field in the Standard Model may have a second degenerate minimum at an ultrahigh vacuum expectation value. This second minimum then determines, by radiative corrections, the values of the top-quark and Higgs-boson masses at the standard minimum corresponding to the electroweak energy scale. An argument is presented that this ultrahigh vacuum expectation value is proportional to the energy scale of gravity, $E_{\text{Planck}} \equiv \sqrt{\hbar c^5/G_N}$, considered to be characteristic of a spacetime foam. In the context of a simple model, the existence of kink-type wormhole solutions places a lower bound on the ultrahigh vacuum expectation value and this lower bound is of the order of E_{Planck} .

1. INTRODUCTION

The ATLAS and CMS collaborations have recently reported results [1, 2] which appear to confirm the existence of the Higgs boson of the Standard Model with a mass around 126 GeV/ c^2 . Long before that, Froggatt and Nielsen [3] gave a remarkable prediction of the Higgs mass value ($M_{\text{Higgs}} = 135 \pm 9 \text{ GeV}/c^2$) based on a heuristic physical argument, multiple-point criticality. A crucial ingredient of the prediction was the following identification for the ultrahigh vacuum expectation value at a second degenerate minimum of the effective Higgs potential:

$$\phi_{\text{vac},2} \stackrel{?}{\sim} E_{\text{Planck}} \equiv \sqrt{\hbar c^5/G_N} \approx 1.22 \times 10^{19} \text{ GeV}, \quad (1)$$

with the first minimum corresponding to the standard electroweak scale, $\phi_{\text{vac},1} \approx 246 \text{ GeV}$.

But the physics motivation for the identification (1) was indirect and (1) was really only an assumption. In fact, Froggatt and Nielsen did not calculate gravitational effects (governed by Newton's coupling constant G_N) but simply appealed to the relevance of Planckian units as a *deus ex machina*. Obviously, it would be conceptually important to understand why $\phi_{\text{vac},2} \propto 1/\sqrt{G_N}$ and to see that the proportionality constant in (1) is indeed of order 1. Related issues have also been addressed in several recent papers (see, e.g., Refs. [4, 5, 6]), but our approach is different.

It is expected (but, of course, not proven) that the fundamental structure of spacetime changes radically at energies of order E_{Planck} or length scales of the order of $\hbar c/E_{\text{Planck}}$. Over the years, various aspects of this radical change have been considered, going under the names of spacetime foam, superstrings, and loop quantum gravity. As the aim of the present paper is rather

modest, we will restrict ourselves to a very simple model and a very simple calculation. Hopefully, this will give us at least one physical argument of what may determine the parametric dependence of $\phi_{\text{vac},2}$.

2. SETUP

In place of the full Standard Model at typical interaction energies and renormalization scale of order E_{Planck} (described in part by the effective potential [7, 8, 9, 10, 11, 12]), consider a simple classical theory with a single scalar field and Einstein gravity. [Time scales of the classical theory, or length scales divided by c , are converted into inverse-energy scales by the introduction of the Planck constant \hbar .] Concretely, take

- a real classical scalar field $\phi(x)$;
- a scalar potential $V(\phi) \geq 0$ with two degenerate minima, $V(v_1) = V(v_2) = 0$;
- a conformal coupling of the scalar field to gravity (coupling constant $\xi = 1/6$).

Our goal, now, is to perform a toy-model calculation of something like a spacetime foam. The easiest calculation is to look for permanent static Lorentzian wormholes [13]. For the simple classical theory considered, Sushkov and Kim [14] have indeed found regular kink-type wormhole solutions. Remarkably, these solutions only occur for the case of ‘small’ v_1 and ‘large’ v_2 :

$$|v_1| < E_{\text{Planck}}/\sqrt{8\pi\xi}, \quad |v_2| > E_{\text{Planck}}/\sqrt{8\pi\xi}, \quad (2)$$

which can be written more compactly in terms of the so-called reduced Planck energy, $E_P \equiv E_{\text{Planck}}/\sqrt{8\pi}$. The heuristic explanation of (2) is that for this case

¹⁾ e-mail: frans.klinkhamer@kit.edu

the conformal factor $f(\phi) \equiv 1 - 8\pi\xi\phi^2/(E_{\text{Planck}})^2$ of a kink-type scalar field configuration $\phi(\rho)$ can vanish for one and only one value ρ_0 of the radial coordinate ρ , whereas pairs of such points, $\rho_{0,1}$ and $\rho_{0,2}$, would have a nondifferentiable solution in between [this conformal factor $f(\phi)$ multiplies the Ricci scalar R in the action and further details can be found below].

Note the crucial role of having finite positive ξ in (2) and the possibly convincing argument in favor of the value $\xi = 1/6$ from conformal symmetry (see, e.g., the discussion in Ref. [4]).

But before investigating the implications of (2) for the electroweak theory, we must make sure that a wormhole solution still exists if $|v_1| \ll |v_2|$.

3. MODEL

We consider the following classical model (setting $G_N = c = \hbar = 1$ and using the same conventions as in Ref. [14]):

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi} R - \frac{1}{2} g^{\mu\nu} \phi_{;\mu} \phi_{;\nu} - \frac{1}{2} \xi \phi^2 R - V(\phi) \right], \quad (3a)$$

$$V(\phi) = \frac{\lambda}{4} (\phi - v_1)^2 (\phi - v_2)^2, \quad (3b)$$

$$\lambda > 0, \quad 0 \leq v_1 < v_2, \quad (3c)$$

$$\xi = 1/6. \quad (3d)$$

A more realistic potential would involve logarithms of ϕ^2 (cf. Refs. [7, 8, 9]), but the polynomial potential (3b) is used for simplicity.

Following Ref. [14], the spherically symmetric static *Ansatz* is given by

$$ds^2 = -A(\rho) dt^2 + \frac{d\rho^2}{A(\rho)} + \hat{r}^2(\rho) (d\theta^2 + \sin^2 \theta d\phi^2), \quad (4a)$$

$$\phi = \phi(\rho). \quad (4b)$$

At this moment, it turns out to be useful to introduce further model parameters:

$$\kappa \equiv m/\sqrt{\lambda} \equiv (v_2 - v_1)/2, \quad (5a)$$

$$\bar{\phi} = (v_2 + v_1)/2, \quad (5b)$$

and the following dimensionless variables:

$$y \equiv \frac{m\rho}{\sigma + |m\rho|}, \quad (6a)$$

$$r(y) \equiv m\hat{r}(\rho), \quad (6b)$$

$$\eta(y) \equiv \phi(\rho)/\kappa, \quad (6c)$$

$$\bar{\eta} \equiv \bar{\phi}/\kappa, \quad (6d)$$

with a positive numerical constant σ in the definition of the compactified dimensionless radial coordinate y . The minima of the potential (3b) then occur for the following vacuum expectation values of the dimensionless scalar field:

$$\eta_1 = \bar{\eta} - 1, \quad \eta_2 = \bar{\eta} + 1. \quad (7)$$

For the above *Ansatz* and definitions, the reduced field equations and boundary conditions are given by Eqs. (4.34)–(4.39) in Ref. [14], where a typo in the definition of f stands to be corrected. These reduced field equations can only be solved numerically.

4. NUMERICAL SOLUTION

The authors of Ref. [14] have presented a numerical solution (also reproduced by us) for a particular set of parameters and boundary conditions, having, in particular, scalar minima $\eta_1 \approx 1.4495$ and $\eta_2 \approx 3.4495$ for model parameter $\bar{\eta} = \sqrt{6}$. The corresponding dimensional vacuum expectation values v_1 and v_2 are both Planckian, whereas we are interested in having one, v_1 , at the electroweak scale.

We have, therefore, obtained a numerical solution for $\eta_1 = 0$ and $\eta_2 = 2$; see the caption of Fig. 1 for the specific parameters and boundary conditions used (the conformal factor $1 - 8\pi\xi\kappa^2\eta^2$ vanishes at $y = 0$). The resulting spacetime and scalar field configuration (Fig. 1) can be described as follows:

- on the ‘outside’ of the wormhole ($y > y_{\text{throat}} \approx -0.16$), there is a smooth approach to the standard Minkowski spacetime and the Standard Model Higgs vacuum $\phi = v_1$.
- on the ‘inside’ of the wormhole ($y < y_{\text{throat}}$), there is a Planck-scale scalar field $\phi \sim v_2$ with effective energy densities of order $-(E_{\text{Planck}})^4$ close to the wormhole throat, which may be viewed as a caricature of what a dynamical quantum spacetime foam can look like at ultrashort length scales.

The results shown in Fig. 1 can be expected to give a reasonably accurate approximation of the exact wormhole-type solution over the coordinate interval $-0.5 \lesssim y \lesssim 0.75$.

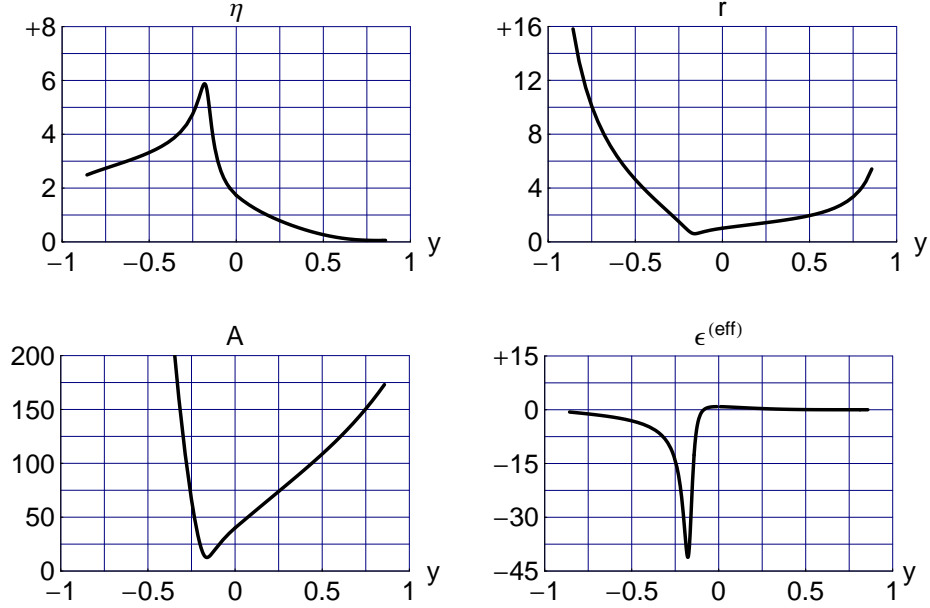


Fig. 1: Numerical solution of the reduced field equations from the spherically symmetric *Ansatz* (4) in the scalar-gravity model (3). The top-left panel shows the dimensionless scalar field η as a function of the compactified dimensionless radial coordinate y . The scalar field $\eta(y)$ is seen to interpolate between the vacuum values $\eta_1 = 0$ and $\eta_2 = 2$, making for a non-monotonic kink-type configuration. The further panels show the dimensionless metric functions $A(y)$ and $r(y)$ and the dimensionless effective energy density $\epsilon^{(\text{eff})}[\eta(y), A(y), r(y)]$. The dimensionless ‘radius’ function $r(y)$ is seen to have a minimum value of approximately 0.6 at radial coordinate $y \approx -0.16$, which corresponds to the so-called wormhole throat [13]. The model parameters used are $\{\xi, \lambda, \kappa, \bar{\eta}, \sigma\} = \{1/6, 1, 1/(2\sqrt{\pi}), 1, 10\}$. For the numerical solution of the reduced field equations [three second-order ordinary differential equations], the boundary conditions at $y = 0$ are $\{r(0), A(0), \eta(0), \eta'(0)\} = \{1017/1000, 40, \sqrt{3}, -3/5\}$, with $r'(0)$ and $A'(0)$ values from two constraint equations (see Ref. [14] for details).

5. DISCUSSION

In view of these numerical results and in line with condition (2), the first wormhole solutions of model (3) would occur for

$$v_1 \ll E_{\text{Planck}}, \quad (8a)$$

$$\begin{aligned} v_2 &\sim \sqrt{\frac{1}{8\pi\xi}} E_{\text{Planck}} = \frac{1}{2} \sqrt{\frac{3}{\pi}} E_{\text{Planck}} \\ &\approx 5.97 \times 10^{18} \text{ GeV}, \end{aligned} \quad (8b)$$

where the conformal value (3d) for the coupling constant ξ has been used in the last step. Now, this is indeed what may be relevant for the renormalization-group-improved effective potential of the Standard Model [7, 8, 9] entering the multiple-point-criticality argument of Froggatt and Nielsen [3], with $v_1 \sim 10^2$ GeV and $v_2 \sim 10^{19}$ GeV.

Taking (8b) at face value and extrapolating one set of NNLO results from the right panel of Fig. 4 in Ref. [12] gives the following pole masses: $M_{\text{Higgs}} =$

126 GeV and $M_{\text{top}} \approx 171.4$ GeV, for $\alpha_s(M_Z) = 0.1184$. With input values $M_{\text{Higgs}} \in [124 \text{ GeV}, 128 \text{ GeV}]$ and $\alpha_s(M_Z) \in [0.1160, 0.1210]$, there is the following prediction by linear approximation: $M_{\text{top}}[\text{GeV}] \approx 171.4 + (M_{\text{Higgs}}[\text{GeV}] - 126)/2 + (\alpha_s(M_Z) - 0.1184)/0.0028$, with an estimated theoretical 1σ uncertainty of ± 0.5 (see Ref. [12] for details and further discussion of technical issues).

We repeat that the simple classical model (3) is only considered to describe certain aspects of the Standard Model physics at typical interaction energies and renormalization scale of order E_{Planck} (observe, for example, that the curvature around the v_1 minimum has a Planckian order of magnitude, contrary to what is observed experimentally [1, 2]). Still, the Standard Model fields may suffice to explain all particle physics results known to date, including neutrino masses and mixing (the dimension-5 term discussed in Ref. [15] would have a mass scale $M_5 \sim v_2/c^2$).

Let us close with two remarks. First, there is, in principle, no problem to extend the *Ansatz* (4) of

the simple model to the Standard Model fields, having made an obvious generalization of the degenerate potential (3b) and adding appropriate spherically symmetric gauge fields. Assuming that a regular solution exists, the next issue is stability. We are moderately optimistic because the existence and stability of the flat-spacetime kink solution in $1 + 1$ dimensions does not require gauge fields in the first place (different from the Nielsen–Olesen vortex solution in the Abelian $U(1)$ Higgs model, which, in fact, loses its stability when embedded in the Standard Model [16]).

Second, indirect (Cherenkov) experimental bounds [17] require a sufficiently dilute gas of static wormholes as considered in this paper. But, if there exist indeed wormhole-type spacetime defects, they are, most likely, nonstatic and without preferred frame. The simple type of wormhole solution considered here is only for the purpose of determining the parametric behavior of the ultrahigh vacuum expectation value of a second degenerate minimum of the effective Higgs potential.

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